Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Notation

$\mathbb{N}$	set of all natural numbers $1, 2, 3, \cdots$
$\mathbb{R}$	set of all real numbers
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of size $m \times n$ with entries in $\mathbb{R}$
$\emptyset$	empty set
$X \setminus Y$	set of all elements from the set X which are not in the set Y
$\mathbb{Z}_n$	group of all congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively
S_n	group of all permutations of the set $\{1, 2, 3, \cdots, n\}$
ln	logarithm to the base e
log	logarithm to the base 10
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$\det(M)$	determinant of a square matrix M

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m} .$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent
(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent
(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent
(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

Q.2 Let $T \in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$ defined by

$$V = \{X \in M_{n\times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of V is

(A) $pn - \text{rank}(T)$ (B) $mn - p\,\text{rank}(T)$
(C) $p(m - \text{rank}(T))$ (D) $p(n - \text{rank}(T))$

Q.3 Let $g: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f: \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = g(x^2 + y^2 - 2z^2).$$

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(A) $4(x^2 + y^2 - 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(B) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(C) $4(x^2 + y^2 - 2z^2)\, g''(x^2 + y^2 - 2z^2)$
(D) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

Q.4 Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n x^n$

(A) converges for all x with $|x| < 2$
(B) converges for all x with $|x| > 2$
(C) does not converge for any x with $|x| > 2$
(D) does not converge for any x with $|x| < 2$

Q.5 Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(A) $\mathbb{Q}_+ \subseteq S$ (B) $S \subseteq \mathbb{Q}_+$
(C) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \emptyset$ (D) $S \cap \mathbb{Q}_+ \neq \emptyset$

Q.6 If $x^h y^k$ is an integrating factor of the differential equation

$$y(1+xy)\,dx + x(1-xy)\,dy = 0,$$

then the ordered pair (h,k) is equal to

(A) $(-2,-2)$ (B) $(-2,-1)$ (C) $(-1,-2)$ (D) $(-1,-1)$

Q.7 If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(A) 1 (B) 4 (C) 5 (D) 9

Q.8 The equation of the tangent plane to the surface $x^2 z + \sqrt{8 - x^2 - y^4} = 6$ at the point $(2,0,1)$ is

(A) $2x + z = 5$ (B) $3x + 4z = 10$
(C) $3x - z = 10$ (D) $7x - 4z = 10$

Q.9 The value of the integral

$$\int_{y=0}^{1} \int_{x=0}^{1-y^2} y \sin\left(\pi(1-x)^2\right) dx\, dy$$

is

(A) $\frac{1}{2\pi}$ (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

Q.10 The area of the surface generated by rotating the curve $x = y^3$, $0 \le y \le 1$, about the y-axis, is

(A) $\frac{\pi}{27} 10^{3/2}$ (B) $\frac{4\pi}{3}(10^{3/2} - 1)$ (C) $\frac{\pi}{27}(10^{3/2} - 1)$ (D) $\frac{4\pi}{3} 10^{3/2}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is

(A) 3 (B) 4 (C) 6 (D) 12

Q.12 Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then

(A) $P^4 = 4P^2 + 9I$ (B) $P^4 = 4P^2 - 9I$ (C) $P^4 = 2P^2 - 9I$ (D) $P^4 = 2P^2 + 9I$

Q.13 The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$, is

(A) connected and compact
(B) connected but not compact
(C) not connected but compact
(D) neither connected nor compact

Q.14 The set $\left\{\frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$, is

(A) compact and open
(B) compact but not open
(C) not compact but open
(D) neither compact nor open

Q.15 For $-1 < x < 1$, the sum of the power series $1 + \sum_{n=2}^{\infty} (-1)^{n-1} n^2 x^{n-1}$ is

(A) $\frac{1-x}{(1+x)^3}$
(B) $\frac{1+x^2}{(1+x)^4}$
(C) $\frac{1-x}{(1+x)^2}$
(D) $\frac{1+x^2}{(1+x)^3}$

Q.16 Let $f(x) = (\ln x)^2$, $x > 0$. Then

(A) $\lim_{x\to\infty} \frac{f(x)}{x}$ does not exist
(B) $\lim_{x\to\infty} f'(x) = 2$
(C) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big) = 0$
(D) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big)$ does not exist

Q.17 Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$. Then $f(1)$ lies in the interval

(A) $(0,\ e^{-1})$
(B) $(e^{-1}, \sqrt{e})$
(C) $(\sqrt{e},\ e)$
(D) (e, ∞)

Q.18 For which one of the following values of k, the equation

$$2x^3 + 3x^2 - 12x - k = 0$$

has three distinct real roots?

(A) 16
(B) 20
(C) 26
(D) 31

Q.19 Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$
(B) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$
(D) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

Q.20 Let S be the family of orthogonal trajectories of the family of curves

$$2x^2 + y^2 = k, \text{ for } k \in \mathbb{R} \text{ and } k > 0.$$

If $\mathcal{C} \in S$ and $\mathcal{C}$ passes through the point $(1, 2)$, then $\mathcal{C}$ also passes through

(A) $(4, -\sqrt{2})$ (B) $(2, -4)$ (C) $(2, 2\sqrt{2})$ (D) $(4, 2\sqrt{2})$

Q.21 Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(A) $e + 1$ (B) $2e + 3$ (C) $3e + 2$ (D) $3e + 1$

Q.22 The function

$$f(x, y) = x^3 + 2xy + y^3$$

has a saddle point at

(A) $(0, 0)$ (B) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$ (C) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (D) $(-1, -1)$

Q.23 The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

(A) $\frac{\pi}{2}(17^{3/2} - 1)$ (B) $\pi(17^{3/2} - 1)$ (C) $\frac{\pi}{6}(17^{3/2} - 1)$ (D) $\frac{\pi}{3}(17^{3/2} - 1)$

Q.24 Let $\mathcal{C}$ be the circle $(x - 1)^2 + y^2 = 1$, oriented counter clockwise. Then the value of the line integral

$$\oint_{\mathcal{C}} -\frac{4}{3}xy^3 \, dx + x^4 \, dy$$

is

(A) 6π (B) 8π (C) 12π (D) 14π

Q.25 Let $\vec{F}(x, y, z) = 2y\,\hat{\imath} + x^2\,\hat{\jmath} + xy\,\hat{k}$ and let $\mathcal{C}$ be the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Then the value of

$$\left| \oint_{\mathcal{C}} \vec{F} \cdot d\vec{r} \right|$$

is

(A) π (B) $\frac{3\pi}{2}$ (C) 2π (D) 3π

Q.26 The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point $(1, 1, 2)$ passes through

(A) $(-1, -2, 4)$ (B) $(-1, 4, 4)$ (C) $(3, 4, 4)$ (D) $(-1, 4, 0)$

Q.27 The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Q.28 Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if the series

(A) $\sum_{n=1}^{\infty} a_n^2$ converges
(B) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges
(C) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges
(D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

Q.29 For $\beta \in \mathbb{R}$, define

$$f(x, y) = \begin{cases} \dfrac{x^2 |x|^{\beta} y}{x^4 + y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then, at $(0, 0)$, the function f is

(A) continuous for $\beta = 0$
(B) continuous for $\beta > 0$
(C) not differentiable for any β
(D) continuous for $\beta < 0$

Q.30 Let $\{a_n\}$ be a sequence of positive real numbers such that

$$a_1 = 1, \ a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(A) $(1, 2]$ (B) $(2, 3]$ (C) $(3, 4]$ (D) $(4, 5]$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$
II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_8$ to G

Then

(A) I is true
(B) I is false
(C) II is true
(D) II is false

Q.32 Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by

$$f(x) = yxy^{-1},\ g(x) = x^{-1} \text{ and } h = g \circ g.$$

Then

(A) g and h are homomorphisms and f is not a homomorphism
(B) h is a homomorphism and g is not a homomorphism
(C) f is a homomorphism and g is not a homomorphism
(D) f, g and h are homomorphisms

Q.33 Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then

(A) rank(T) $\geq$ nullity(S)
(B) rank(S) $\geq$ nullity(T)
(C) rank(T) $\leq$ nullity(S)
(D) rank(S) $\leq$ nullity(T)

Q.34 Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then

(A) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$
(B) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$
(C) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} - \nabla g \cdot \vec{F}$
(D) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} + \nabla g \cdot \vec{F}$

Q.35 Consider the intervals $S = (0, 2]$ and $T = [1, 3)$. Let S° and T° be the sets of interior points of S and T, respectively. Then the set of interior points of $S \setminus T$ is equal to

(A) $S \setminus T^\circ$
(B) $S \setminus T$
(C) $S^\circ \setminus T^\circ$
(D) $S^\circ \setminus T$

Q.36 Let $\{a_n\}$ be the sequence given by

$$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, \quad n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?

(A) Both the subsequences are convergent
(B) Only one of the subsequences is convergent
(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$
(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

Q.37 Let

$$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x| \text{ and } g(x) = x^2 \text{ for } x \in \mathbb{R}.$$

If $h(x) = f\big(g(x)\big)$, then

(A) h is not differentiable at $x = 0$
(B) $h'\big(\sqrt{\pi}\big) = 0$
(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$
(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q.38 Let $f: \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by

$$f(x) = (\sin x)^{\pi} - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

(A) f is an increasing function
(B) f is a decreasing function
(C) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$
(D) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

Q.39 Let

$$f(x,y) = \begin{cases} \dfrac{|x|}{|x| + |y|}\sqrt{x^4 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Then at $(0,0)$,

(A) f is continuous
(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist
(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$
(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Q.40 Let $\{a_n\}$ be the sequence of real numbers such that

$$a_1 = 1 \text{ and } a_{n+1} = a_n + a_n^2 \text{ for all } n \geq 1.$$

Then

(A) $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$
(B) $\lim_{n\to\infty} \frac{1}{a_n} = 0$
(C) $\lim_{n\to\infty} \frac{1}{a_n} = 1$
(D) $\lim_{n\to\infty} a_n = 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let x be the 100-cycle $(1\ \ 2\ \ 3 \cdots 100)$ and let y be the transposition $(49\ \ 50)$ in the permutation group S_{100}. Then the order of xy is ______

Q.42 Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$$W_1 = \{\, (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 4\},$$

$$W_2 = \{\, (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 5\}.$$

Then the dimension of $W_1 \cap W_2$ is _____

Q.43 Consider the following system of three linear equations in four unknowns x_1, x_2, x_3 and x_4

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4, \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 5, \\ x_1 + 3x_2 + 5x_3 + kx_4 &= 5. \end{aligned}$$

If the system has no solutions, then $k =$ ________

Q.44 Let $\vec{F}(x,y) = -y\,\hat{\imath} + x\,\hat{\jmath}$ and let C be the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

oriented counter clockwise. Then the value of $\oint_C \vec{F} \cdot d\vec{r}$ (round off to 2 decimal places) is________________

Q.45 The coefficient of $\left(x - \frac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \dfrac{4(1 - \sin x)}{2x - \pi}, & x \neq \dfrac{\pi}{2} \\ 0, & x = \dfrac{\pi}{2} \end{cases}$$

about $x = \frac{\pi}{2}$, is _____________

Q.46 Let $f: [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \frac{\left(1+x^{\frac{1}{3}}\right)^3 + \left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$

Then

$$\max\{f(x): x \in [0,1]\} - \min\{f(x): x \in [0,1]\}$$

is ___________

Q.47 If

$$g(x) = \int_{x(x-2)}^{4x-5} f(t)\,dt, \text{ where } f(x) = \sqrt{1+3x^4} \text{ for } x \in \mathbb{R}$$

then $g'(1) =$ _______

Q.48 Let

$$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2-y^2}, & x^2-y^2 \neq 0 \\ 0, & x^2-y^2 = 0. \end{cases}$$

Then the directional derivative of f at $(0, 0)$ in the direction of $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ is __________

Q.49 The value of the integral

$$\int_{-1}^{1}\int_{-1}^{1} |x+y|\,dx\,dy$$

(round off to 2 decimal places) is ___________

Q.50 The volume of the solid bounded by the surfaces $x = 1 - y^2$ and $x = y^2 - 1$, and the planes $z = 0$ and $z = 2$ (round off to 2 decimal places) is _____________

Q. 51 – Q. 60 carry two marks each.

Q.51 The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (round off to 2 decimal places) is ___________

Q.52 The greatest lower bound of the set

$$\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\},$$

(round off to 2 decimal places) is _______________

Q.53 Let $G = \{n \in \mathbb{N} : n \leq 55, \ \gcd(n, 55) = 1\}$ be the group under multiplication modulo 55. Let $x \in G$ be such that $x^2 = 26$ and $x > 30$. Then x is equal to_____________

Q.54 The number of critical points of the function

$$f(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$$

is ____________

Q.55 The number of elements in the set $\{x \in S_3 : x^4 = e\}$, where e is the identity element of the permutation group S_3, is ____________

Q.56 If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$, $y, z \in \mathbb{R}$, is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$ then $z - y$ is equal to___________

Q.57 Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then the maximum value of $\det(M) + \det(N)$ is ____________

Q.58 Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to__________

Q.59 Let $y(x) = xv(x)$ be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0.$$

If $v(0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to_____________

Q.60 If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0,\ y(0) = 2,\ \frac{dy}{dx}(0) = 0,$$

then $y(\ln 2)$ is (round off to 2 decimal places) equal to ____________

END OF THE QUESTION PAPER